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The *q*-factor and the Fama and French asset pricing models: hedge fund evidence

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Abstract

Purpose – The purpose of this paper is to test the new Fama and French (2015) five-factor model relying on a thorough sample of hedge fund strategies drawn from the Barclay's Global hedge fund database.
Design/methodology/approach – The authors use a stepwise regression to identify the factors of the *q*-factor model which are relevant for the hedge fund strategy analysis. Doing so, the authors account for the Fung and Hsieh seven factors which prove very useful in the explanation of the hedge fund strategies. The authors introduce interaction terms to depict any interaction of the traditional Fama and French factors with the factors associated with the *q*-factor model. The authors also examine the dynamic dimensions of the risk-taking behavior of hedge funds using a BEKK procedure and the Kalman filter algorithm.
Findings – The results show that hedge funds seem to prefer stocks of firms with a high investment-to-assets ratio (low conservative minus aggressive (CMA)), on the one hand, and weak firms' stocks (low robust minus weak (RMW)), on the other hand. This combination is not associated with the conventional properties of growth stocks – i.e., low high minus low (HML) stocks – which are related to firms which invest more (low CMA) and which are more profitable (high RMW). Finally, small minus big (SMB) interacts more with RMW while HML is more correlated with CMA. The conditional correlations between SMB and CMA, on the one hand, and HML and RMW, on the other hand, are less tight and may change sign over time.
Originality/value – To the best of the authors' knowledge, the authors are the first to cast the new Fama and French five-factor model in a hedge fund setting which account for the Fung and Hsieh option-like trading strategies. This approach allows the authors to better understand hedge fund strategies because *q*-factors are useful to study the dynamic behavior of hedge funds.

Keywords Finance, Econometrics, Financial modelling, Stock returns
Paper type Research paper

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1. Introduction

Tobin's q is a device through which financial conditions are channeled to the real sector – especially real investment[1] (Tobin, 1969). It is also the keystone of the q-factor asset pricing model (Cochrane, 1991, 1996). According to this model, real investment is maximized when the marginal benefit of investment – i.e., Tobin's q or the expected discounted cash-flows of investment[2] – is equal to its marginal cost which is associated with the investment expense. In this theory, there are only two drivers of stock expected return: the expected discounted profitability of the firm – Tobin's q – which impacts positively expected return; and the investment-to-assets ratio, which impacts negatively expected return. It is obvious that when the expected profitability of a firm increases, the expected return on its stock also tends to increase. The simple Gordon's theory of asset pricing is based on this relationship (Gordon and Shapiro, 1956)[3]. On the other hand, in line with the well-known Keynesian marginal efficiency of capital schedule[4], a low level of investment is associated with a high cost of capital and, conversely, a high level of investment is associated with a low cost of capital. In other words, when the cost of capital decreases, this tends to stimulate investment. Theoretically, there is thus a negative relationship between a firms' stock expected return (cost of capital) and its level of investment. Moreover, as argued by Cochrane (2008, 2011), aggregate investment is high when stock prices are high – i.e., when expected stock returns are low.

Fama and French (1993) three-factor setting has been used in many studies to model the static dimensions[5] of hedge fund strategy returns (e.g. Capocci and Hübner, 2004; Agarwal and Naik, 2004; Racicot and Théoret, 2012, 2014). Fama and French (2015) have recently revisited their well-known three-factor model (Fama and French, 1993) to incorporate two additional factors – i.e., the two key factors of the q-factor asset pricing model: firms' profitability and the investment-to-assets ratio. This is an important extension of their model since that, while the Fama and French three-factor model does not lie on any precise theoretical model[6], the q-factor factor model is derived from an optimization framework. Cochrane (1991, 1996) has previously been the first to rely on the q-theory to price assets, being in line with the investment-based approach to asset pricing (Abel, 1983). More recently, Hou *et al.* (2015) have also experimented with a new factor model that consists of a market factor, a size factor, an investment factor and a profitability factor – i.e., return on equity. This latter model shares a great similarity with the Fama and French (2015) new five-factor model. The originality of Fama and French's study is to analyze the interactions of their two additional factors (mimicking portfolios) related to the q-factor model – conservative minus aggressive (CMA)[7] and robust minus weak (RMW)[8], the investment and profitability factors, respectively – with their size factor (small minus big (SMB))[9] and their value factor (high minus low (HML))[10], which does not appear in the Hou *et al.*'s (2015) study.

One of the most interesting findings of Fama and French (2015) is that the HML factor is made almost redundant when adding CMA and RMW to the return equation in the sense that RMW and especially CMA seem to capture the risk dimensions of HML: when adding CMA and RMW in their asset pricing model, the factor loading of HML is no longer significant. However, as mentioned by Fama and French (2015), the redundancy of HML may be attributable to their sample and other studies must be achieved before arriving at this conclusion. According to Fama and French (2015): "This result is so striking we caution the reader that it may be specific to this sample".

In line with these developments, our paper proposes to add the investment (CMA) and profitability (RMW) factors in asset pricing models used in the hedge fund industry. In this respect, the value factor (HML) and especially the size factor (SMB) were found quite relevant to explain hedge fund returns in previous studies (Agarwal and Naik, 2004; Capocci and Hübner, 2004; Racicot and Théoret, 2013). Since, according to Fama and French (2015), the CMA and RMW factors are close substitutes to SMB and HML, we aim at studying the interactions of these four factors in the framework of the q-factor asset pricing model (Fama and French, 2015). According to Cochrane (2008, 2011), these two additional factors – i.e., the investment and profitability factors – are more sensitive to macroeconomic time series than the traditional explanatory variables used in factorial models, like the size and value factors. These factors are thus very relevant in a study like ours which aims at analyzing the asymmetrical behavior of hedge fund strategies over the business cycle. Moreover, one of the aims of many hedge fund strategies is to capture the risk premia associated with market anomalies – like the small firm and the value anomalies[11].

In this study, we apply the Fama and French five-factor model to the strategies followed by hedge funds included in the BarclayHedge's database[12] over the period 1997-2015[13]. More precisely, in addition to the general index, we estimate the returns of 18 hedge fund strategies – a rich database for additional tests about the relative contribution of the CMA and RMW factors. We aim at two objectives. First, using stepwise regressions, we investigate whether the factors associated with the q-model shed more light on the strategies followed by hedge funds. In this respect, we account for the option-like trading strategies of hedge funds by adding the Fung and Hsieh's (1997, 2001, 2004) seven factors[14] to the Fama and French five-factor model. Second, we study the interactions between the CMA and RMW factors with the SMB and HML ones. We are particularly interested in the redundancy of HML when adding CMA and RMW to the strategies' return equations. We also test whether HML interacts more with CMA than with RMW as conjectured by Fama and French (2015).

Our empirical results show that the HML factor is quite redundant for most strategies' returns but in several cases, the coefficient of HML remains significant when adding CMA and RMW. The impact of HML in the three-factor model is shared between CMA and RMW. We cannot establish that most of the impact of HML is mostly captured by CMA in the framework of our study even if CMA is closer to HML than RMW. Excepting the short-bias strategy[15], returns of hedge fund strategies have a negative HML loading in the Fama and French three-factor model. But in their five-factor model, HML usually disappears in our stepwise regressions and CMA and RMW usually both substitute for HML with a negative contribution in the strategy return equations. Hedge funds thus tend to prefer growth stocks to value stocks. Or, alternatively in the q-space, they prefer stocks issued by firms with a high investment-to-assets ratio[16] and firms which are less profitable[17] – i.e., which are in the lower quintiles built over return on equity or return on assets. Our tests also suggest a great degree of interaction between the SMB and RMW factors. In this respect, we introduce interaction terms wherever possible in our robustness checks, and they are usually very significant. Thus, our main contribution is to show that there are interactions between the value, size, investment and profitability factors. However, as a rule of thumb, our experiments suggest that HML is closer to CMA and RMW is closer to SMB. These results suggest that small firms tend to be "weak" when compared to big ones – a quite sensitive finding – and that firms which have a high book-to-asset ratio also display a low investment-to-assets ratio, a result supported by Fama and French's (2015) study. However, the link we find between HML and RMW – albeit weaker than the one between HML and CMA – suggests that growth stocks are issued by weak firms. This result is perhaps related to the high rate of bankruptcy observed in the sector of growth stocks (e.g. high-tech, telecommunications stocks) during crises. It is better understood using a dynamic approach rather than a static one. In this respect, CMA and RMW are more cyclical variables than SMB and HML, which are associated with the static (accounting) dimensions of firms. Indeed, investment and profitability, to which CMA and RMW are respectively related, have strong links with aggregate macroeconomic variables (Cochrane, 2008, 2011) and are thus more prone to reflect the reaction of hedge fund strategies over the business cycle.

This paper is organized as follows. Section 2 discusses the Fama and French five-factor asset pricing model. Section 3 presents the data and provides the stylized facts related to the interactions between the factors. Section 4 exposes our methodology and discusses our empirical findings. Section 5 concludes.

2. The *q*-factor model and the Fama and French five-factor model

2.1 Euler equation and the role of investment and profitability

The first order condition (Euler equation) of the q-factor model stipulates that firms will continue to invest until the marginal cost of investment is equal to its marginal benefit – i.e., Tobin's q (Tobin, 1969; Cochrane, 1991, 2011; Hou *et al.*, 2015)[18]:

$$1+a\frac{I_{it}}{A_{it}}=E_t[M_{t+1}\pi_{it+1}] \tag{1}$$

where I_{it} is the investment level of firm i; A_{it} is the level of firm's assets; a is the marginal cost of adjusting the level of capital to its target value; $E_t[.]$ is the expectation operator conditional on the information set available at time t; M_{t+1} is the stochastic discount factor – i.e., $M_{t+1}=\beta(u'(c_{t+1})/u'(c_t))$, where $\beta=(1/1+\rho)$, ρ being the rate of time preference and $u'(c_t)$ is the marginal utility of consumption at time t – and π_{it+1} is the investment cash-flow. The LHS of (1) is the marginal cost of investment and its RHS, the marginal benefit of investment – i.e., Tobin's q (1969).

Equation (1) may be rewritten as follows (Hou *et al.*, 2012, 2015):

$$E\left[r_{i,t+1}\right]=\frac{E_t\left(\pi_{i,t+1}\right)}{1+a\frac{I_{it}}{A_{it}}} \tag{2}$$

According to Equation (2), the expected stock return is related positively to its expected profitability as measured by $E_t(\pi_{i,t+1})$ and negatively to its investment-to-assets ratio, as measured by (I_{it}/A_{it}). This is the essence of the q-factor model.

The Fama and French (2015) five-factor model adds to its three original factors the profitability and investment factors to capture the implications of the q-factor model. It is formulated as follows:

$$R_{it}-r_{ft}=\alpha+\beta_1\left[R_{mt}-r_{ft}\right]+\beta_2 SMB_t+\beta_3 HML_t+\beta_4 UMD_t$$
$$+\beta_5 CMA_t+\beta_6 RMW_t+\varepsilon_{it} \tag{3}$$

where R_{it} is the firm's stock return and r_{ft} is the risk-free rate; R_{mt} is the market portfolio return; SMB is a diversified portfolio which is long in small firms' stocks and short in big firms' stocks; HML is a diversified portfolio which is long in firms whose stocks have a high book-to-market ratio (i.e. value stocks) and which is short in stocks associated with a low book-to-market ratio (i.e. growth stocks); CMA is a diversified portfolio which is long in firms embedded with a low investment-to-assets ratio and short in firms with a high investment-to-assets ratio. Finally, RMW is a diversified portfolio which is long in firms with a high profitability (in terms of net operating revenue to assets or ROE) and short in firms with a low profitability. The addition of the CMA and RMW factors captures the two drivers of expected returns in the q-factor model (Equation (2)). We also add the momentum factor UMD proposed by Carhart (1997) in the return equation as many hedge funds follow momentum-based strategies. UMD is a diversified portfolio which is long in returns of selected stocks having a persistent upward trend and short in stocks displaying a persistent downward trend. A momentum investment strategy is the tendency of an investor to buy and sell stocks based on past returns of the stocks, that is, to buy recent winners and sell recent losers (Bikhchandani and Sharma 2000)[19].

Fama and French (2015) show that, at least at the theoretical level, the CMA and RMW factors substitute for the HML factor. To do so, they rewrite the seminal Miller and Modigliani (1961) equation in terms of the book-to-market ratio[20]:

$$\frac{1}{book-to-market_t} = \frac{\sum_{\tau=1}^{\infty} E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t} \tag{4}$$

where $Y_{t+\tau}$ is earnings for period $t+\tau$; B_t is the equity book value; dB_t is the change in B_t[21], and r is the expected stock return[22]. According to Equation (4), a higher book-to-market ratio implies a higher r. Moreover, a higher book-to-market entails a lower profitability ($Y_{t+\tau}$), all else equal. Finally, a higher book-to-market ratio is related to a lower level of investment (Fama and French, 2006, 2015). Therefore, CMA and RMW substitute for HML.

2.2 Risk premia and the q-factor model

The presence of the market risk premium and SMB, HML and UMD factors in the return Equation (3) may be questionable since they are not direct components of the q-factor model. In this respect, Hou *et al.* (2015) and Fama and French (2015) have not been explicit about the definition of the risk premium in the framework of the q-factor model. This risk premium is found by expanding the RHS of Equation (1)[23]:

$$1 + b\frac{I_{it}}{k_{it}} = \frac{E(\pi_{i,t+1})}{1+r_{ft}} + Cov(M_{t+1}, \pi_{i,t+1}) \tag{5}$$

The first term on the RHS of Equation (5) is the risk-neutral present value of the cash-flow as measured by $\pi_{i,t+1}$. The second term is the risk premium in the framework of the q-factor model – i.e., the covariance between the discount factor and the asset cash-flow. A negative covariance corresponds to a positive risk premium – i.e., it leads to a Tobin's q which is below the risk-neutral present value of the cash-flow[24]. In this case, the cash-flow decreases when $(u'(c_{t+1})/u'(c_t))$ increases, that is when the marginal utility of future consumption increases. Therefore, the cash-flow decreases when consumers need it the most – i.e., in periods of low consumption – which leads investors to require a risk premium on the firm's stock[25]. Since a positive risk premium is associated with a negative covariance in Equation (5), it tends to depress investment.

According to Equation (5), in a stochastic world, expected return is related to the investment-to-assets ratio, to profitability and to other factors which are linked to the risk premium (uncertainty). In the CAPM framework, this covariance is related to the market risk premium, which justifies its introduction in Equation (3). But any other factor related to firms' risk may also be introduced in this equation, like SMB, HML and UMD. Moreover, it must be realized that the CMA and RMW factors are only proxies for the investment and profitability factors: they are measured with errors. In this context, every factor which helps forecast returns has its place in the asset pricing equation. And since the SMB, HML and UMD factors may remain significant even after adding CMA and RMW in the Fama and French (2015) equation, they then remain valuable to price assets. In this respect, an empirical asset pricing kernel must span all the states of nature relevant for the estimation of a stock return. If any other factor spans dimensions of the state space not captured by CMA and RMW, it has a role to play in the return equation.

The definition of the risk premium in the setting of the q-factor model may also be viewed differently. A risk premium is defined by a risk factor and by an exposure to this risk factor. In the CAPM, the risk factor is the market risk premium – as measured by $E(R_m) - r_f$ – and the exposure to this factor is the market β. In the q-factor model as given by Equation (2), the factor which represents risk is $E_t(\pi_{i,t+1})$. Profit is a cyclical variable which is related to macroeconomic and financial shocks (Cochrane, 2011). Since equity is in the denominator of Equation (2), the exposure to this risk is measured by leverage. High leverage signals low investment and higher expected returns (Hou *et al.*, 2012, 2015).

Similarly to the market risk premium, SMB, HML and UMD, the CMA and RMW factors command a positive premium. In this respect, given a firm's profitability, a decrease in I_{it} in Equation (2) leads to an increase in $E(r_{i,t+1})$. This relationship is well documented in the financial literature (e.g. Fama and French, 2006, 2015; Hou *et al.*, 2015). Irving Fisher (1930) has established a negative relationship between return and investment – i.e., the Keynesian marginal efficiency of capital schedule. Moreover, when the cost of capital as measured by r_i is high, the level of investment is low because a high cost of capital is associated with a low NPV, all else equal. Another related interpretation is that investment should be high when expected returns (cost of capital) are low, because stock prices are then high (Cochrane, 2008, 2011). There are thus many justifications for the negative relationship between the expected return and the level of investment. The CMA factor captures this relationship. In other respects, according to Equation (2), firms with a high profitability provide higher expected returns than firms with low profitability. The RMW factor embeds this relationship.

3. Data and stylized facts

3.1 Data

The hedge fund strategies' returns are taken from the database managed by BarclayHedge – one of the biggest hedge fund databases in the world[26]. Barclay's Global hedge fund database contains more than 6,400 hedge funds, funds of funds and CTA. Returns provided by the database are net of fees. The survivorship bias is accounted for in this database, as index returns for periods since 1994 include the defunct funds; moreover the database provides the graveyard of defunct funds and post-1994 hedge fund data are less susceptible of measurement errors than pre-1994 data. The data set runs from January 1997 to February 2015, for a total of 218 observations[27]. In addition to the weighted composite return, our sample includes 18 return series of well-known hedge fund strategies. Finally, the Fama and French factors are drawn from French's database[28] and the Fung and Hsieh lookback straddles come from Hsieh's database[29].

3.2 Stylized facts

3.2.1 Interactions between the factors included in the Fama and French five-factor model. Figure 1 provides the conditional correlations between Fama and French factors built using the multivariate GARCH procedure (BEKK procedure: Bollerslev *et al.*, 1988; Engle and Kroner, 1995)[30]. We note that the conditional correlation between SMB and RMW is usually negative and it often exceeds 0.5 in absolute value. The size and the profitability factors thus go in opposite direction. This suggests that small firms tend to be less profitable than big ones. The positive conditional correlation between HML and CMA also tends to be high and persistent[31]. Firms with a high book-to-market ratio – which are associated with value stocks – thus tend to invest less than firms with a low book-to-market ratio (growth stocks). The other correlations between factors – i.e., between SMB and CMA, HML and RMW, and CMA and RMW – are quite loose. More precisely, inside our sample, the theoretical negative conditional correlation between HML and RMW as given by Equation (4) has only been observed since 2007, which is

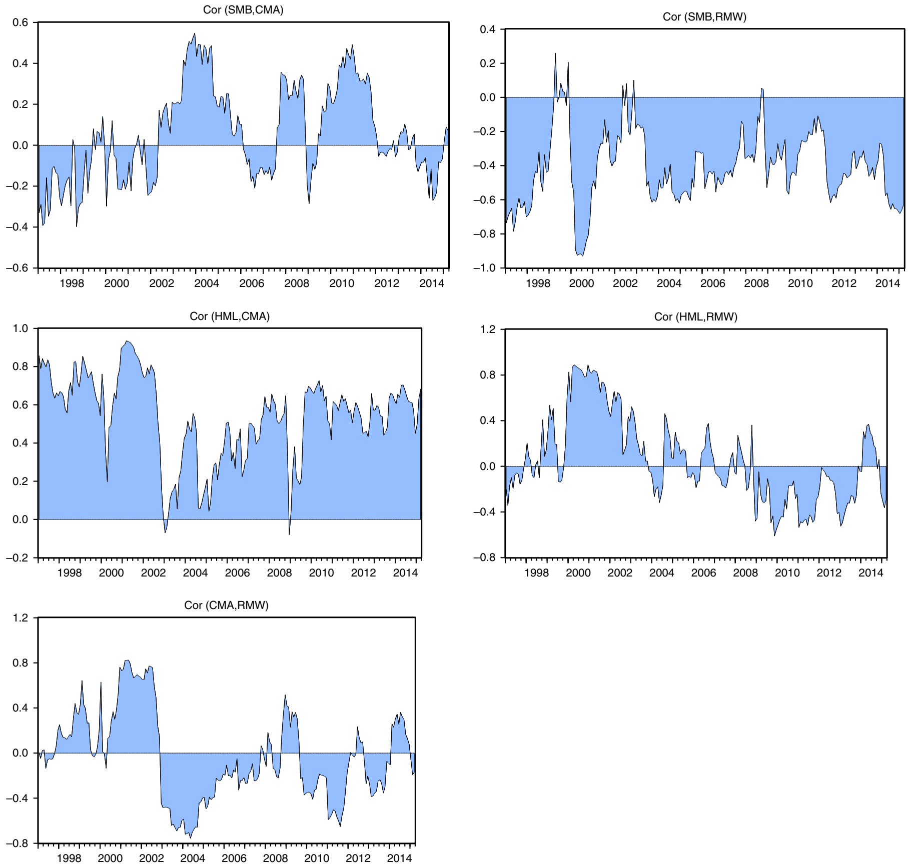


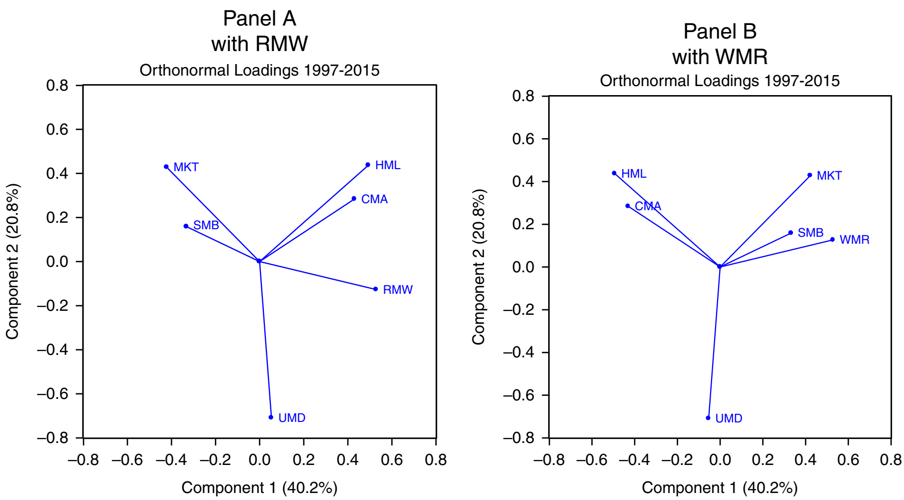
Figure 1.
Conditional correlations between Fama and French factors

Notes: The conditional correlations are computed using the multivariate GARCH (MGARCH) procedure (BEKK specification; Bollerslev *et al.*, 1988; Engle and Kroner, 1995) associated with the start of the subprime crisis[32]. This is an important fact for the interpretation of our findings. In this respect, our dynamic approach allows a better understanding of the Fama and French's puzzle – i.e., small firms that invest a lot despite poor financial results (Fama and French, 2015). The static implications given by Equation (4) are not necessarily true in a dynamic setting. At the least, they require some qualifications.

A look at the plot of orthonormal loadings of the Fama and French factors computed with the principal components analysis confirm these correlations. According to Figure 2 (Panel A), CMA and HML are close factors while the respective positions of RMW and SMB in the plot indicate that the correlation between them is negative and high in absolute value. To better grasp the link between SMB and RMW, we can rotate this last factor multiplying it by −1. We obtain WMR, a portfolio which is long in weak firms and short in robust ones. This rotation is shown in Panel B. Consistent with the conditional correlation, SMB is now very close to WMR. Not surprisingly, SMB is also close to the market factor while UMD stands alone.

We can get a better understanding of the links between factors by running Granger causality tests (Table I). In line with our previous results, the Granger test between CMA and HML is significant in both directions: HML Granger causes CMA, and CMA Granger causes HML, the latter test being more significant than the former. In other respects, RMW also Granger causes HML. However, HML does not Granger causes RMW. Surprisingly, SMB Granger causes CMA and RMW, suggesting that the link between SMB and the two new factors is quite tight. Size does impact investment and profitability levels.

Summarizing, our stylized facts show that the factors related to the *q*-factor asset pricing model – i.e., CMA and RMW – strongly interact with SMB and HML. However, our tests suggest that CMA is closer to HML, and that RMW has a strong negative relationship with SMB. These results are important for the



Notes: Orthonormal loadings are computed using the principal components procedure. WMR=−RMW

Figure 2.
Factors' orthonormal loadings

Test	Statistics	
HML Granger causes (GC) CMA	F-statistic	1.72
	p-value	0.06*
CMA GC HML	F-statistic	1.93
	p-value	0.03**
HML GC RMW	F-statistic	1.21
	p-value	0.27
RMW GC HML	F-statistic	2.27
	p-value	0.01**
SMB GC CMA	F-statistic	2.51
	p-value	0.01**
CMA GC SMB	F-statistic	0.78
	p-value	0.66
SMB GC RMW	F-statistic	3.09
	p-value	0.01**
RMW GC SMB	F-statistic	1.46
	p-value	0.14

Notes: When the test accepts the hypothesis of causality; *,**Significant at 10 and 5 percent levels, respectively

Table I.
Granger causality tests

interpretation of our empirical results, which will be presented in the following section. In contrast to Fama and French (2015), our preliminary results do not allow to conclude that HML is redundant, even if it is strongly linked to RMW and especially to CMA.

3.2.2 Correlations between strategies' returns and factors. Table II provides the correlations between strategies' returns and the four factors associated with the firms in which hedge funds invest: HML, CMA, RMW, and SMB. The hedge fund index and nine strategies are significantly correlated with all of these factors. Except for short bias which follows a contrarian strategy, the correlations between strategies' returns and HML, CMA and RMW are negative, and correlations with SMB are positive, the mean values being −0.11, −0.13, −0.27 and 0.22, respectively. Hedge funds thus prefer to invest in[33]:

(1) firms having a low book-to-market ratio (negative HML);

(2) firms with a high ratio of investment-to-assets (negative CMA);

(3) weak firms in terms of profitability (negative RMW); and

(4) small firms (positive SMB).

The average exposure of hedge funds to SMB and RMW is higher, in absolute value, than their exposure to HML and CMA. Short-bias' returns display a high sensitivity to the four factors, the sign of these sensitivities being the opposite of the other strategies.

The strategies' correlations which are the most related are between strategies' returns and SMB, on the one hand, and strategies' returns and RMW, on the other hand: the more a strategy's return is positively correlated to SMB, the more it is negatively correlated to RMW. In this respect, the scatter diagram appearing in Figure 3 links these two sets of correlations (Panel A). The negative relationship between the correlations associated with these two factors is very tight, the

Correlation Probability	HML	CMA	RMW	SMB
Convert. Arb.	−0.01	−0.08	−0.25	0.20
	0.90	*0.23*	*0.00*	*0.00*
CTA	0.06	0.11	0.07	−0.03
	0.40	*0.11*	*0.30*	*0.65*
Currency	0.10	0.18	0.02	−0.01
	0.14	*0.01*	*0.74*	*0.88*
Distressed	−0.01	−0.11	−0.38	0.37
	0.90	*0.11*	*0.00*	0.00
Diversified	0.05	0.11	0.06	−0.02
	0.45	*0.10*	*0.36*	*0.75*
Emerging	−0.18	−0.29	−0.35	0.29
	0.01	*0.00*	*0.00*	*0.00*
Equity-long	−0.27	−0.36	−0.53	0.43
	0.00	*0.00*	*0.00*	*0.00*
Event_driven	−0.12	−0.21	−0.43	0.40
	0.09	*0.00*	*0.00*	*0.00*
Fixed income	0.03	−0.08	−0.21	0.12
	0.66	*0.24*	*0.00*	*0.08*
Funds of funds	−0.23	−0.25	−0.45	0.36
	0.00	*0.00*	*0.00*	*0.00*
Health care	−0.51	−0.30	−0.74	0.58
	0.00	*0.00*	*0.00*	*0.00*
Long-short	−0.35	−0.34	−0.58	0.46
	0.00	*0.00*	*0.00*	*0.00*
Macro	−0.20	−0.22	−0.30	0.27
	0.00	*0.00*	*0.00*	*0.00*
Market neutral	−0.10	−0.09	−0.09	0.12
	0.13	*0.21*	*0.21*	*0.08*
Merger	−0.08	−0.14	−0.24	0.25
	0.25	*0.04*	*0.00*	*0.00*
Multistrat	−0.07	−0.12	−0.34	0.29
	0.29	*0.08*	*0.00*	*0.00*
Short bias	0.43	0.38	0.64	−0.51
	0.00	*0.00*	*0.00*	*0.00*
Techno	−0.60	−0.49	−0.71	0.45
	0.00	*0.00*	*0.00*	*0.00*
Mean of correlations	*−0.11*	*−0.13*	*−0.27*	*0.22*
gi	−0.25	−0.32	−0.52	0.41
	0.00	*0.00*	*0.00*	*0.00*

Notes: *gi* is the hedge fund general index. The hedge fund strategies' returns are drawn from the Barclay's Global hedge fund database. For each strategy, the first line provides correlation with factors at the head of the columns, and the second line gives the *p*-value of the correlation (in italics)

Table II.
Correlation between strategies' returns and factors

observed correlations being close to the fitted regression line. The regression fit between the correlations of returns associated with HML and CMA is less good (Figure 3, Panel B).

4. Empirical results

4.1 Estimation of the Fama and French three-factor model

Before analyzing the factor interaction in the Fama and French (2015) five-factor model, we first establish the relative weight of factors in the estimated three-factor model using our sample of hedge fund strategies. We add to these three factors the

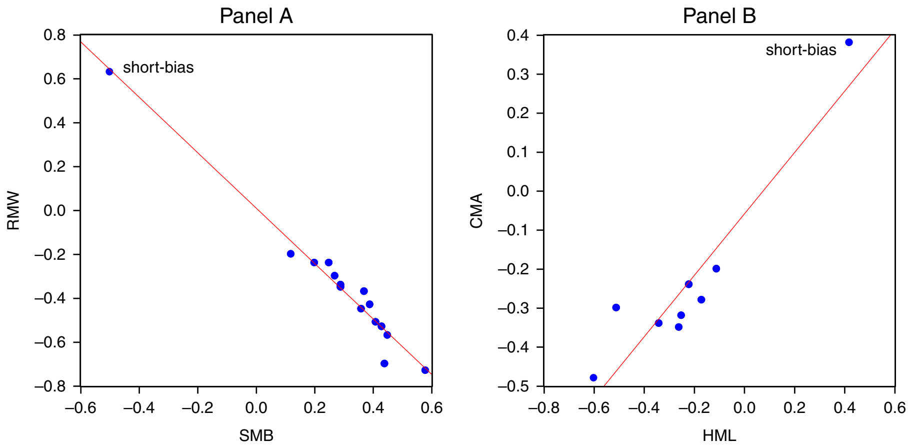


Figure 3.
Scatter diagram of correlations of strategy returns with SMB and RMW, and with HML and CMA

Notes: In panel A, we plot the scatter diagram of the {correlation(r_{it}, SMB_t)} set and the {correlation(r_{it}, RMW_t)} set, r_i being a strategy return. In panel B, we plot the scatter diagram of the {correlation(r_{it}, HML_t)} set and {correlation(r_{it}, CMA_t)} set

momentum factor (UMD) as many hedge funds follow momentum-based strategies. Our first version of the Fama and French (1993) reads as follows:

$$R_{it} - r_{ft} = \alpha + \beta_1 \left[R_{mt} - r_{ft}\right] + \beta_2 \mathrm{SMB}_t + \beta_3 \mathrm{HML}_t$$

$$+ \beta_4 \mathrm{UMD}_t + \beta_5 ar(1) + \varepsilon_{it} \qquad (6)$$

We include an *ar*(1) term in our estimation process to account for the autocorrelation of order 1 between returns. If markets are efficient, there should be no autocorrelation between returns because otherwise they become predictable. However, in the hedge fund industry, autocorrelation may be due to return smoothing or to infrequent trading giving rise to illiquidity (Okunev and White, 2003; Pástor and Stambaugh, 2003; Getmansky *et al.*, 2004; Chan *et al.*, 2007; Brown *et al.*, 2012; Chen and Tindall, 2012). This autocorrelation contributes to obscuring the effective risk in the hedge fund industry. Moreover, it gives rise to estimation biases if not accounted for. We rely on an *ar*(1) process to tackle autocorrelation created by return smoothing or illiquidity in our estimations (Okunev and White, 2003; Bali *et al.*, 2014)[34].

Despite its parsimony, the Fama and French three-factor model performs well in explaining the hedge fund strategies' returns. For the general index (*gi*), the adjusted R^2 is equal to 0.77 and it exceeds 0.50 for 10 strategies (over 18) (Table III). However, the R^2 is lower than 0.10 for three strategies – i.e., CTA, currency and diversified – and, as expected, it is also very low for the fixed income strategy (0.28). Note also the high degree of return autocorrelation in the hedge fund industry as given by the coefficient of *ar*(1). If financial markets were perfect – i.e., without frictions like return smoothing or illiquidity – the return autocorrelation would be zero. For *gi*, the estimated autocorrelation coefficient is equal to 0.32, significant at the 1 percent level. It exceeds 0.40 for the three following strategies: convertible, distressed and multistrategy.

The estimations do not signal any serious α puzzle. At 0.33 percent monthly, or 3.92 percent annually, the α estimated for *gi* is not particularly high. The mean α computed

Table III. Estimation of the conventional Fama and French three-factor model

Dep. Var.	Convertible	CTA	Currency	Distressed	Diversified	Emerging	Equity long	Event driven	Fixed_income	Funds of funds
C	0.3785	0.1826	0.0673	0.3744	0.2990	0.1929	0.2687	0.3826	0.2523	0.1184
	1.63	*1.50*	*0.79*	*2.03*	*1.68*	*0.65*	*2.53*	*3.09*	*1.55*	*1.07*
mkt_rf	0.0918	−0.0242	0.0121	0.1966	−0.0764	0.5844	0.6024	0.2696	0.0931	0.2029
	2.89	*−0.62*	*0.59*	*6.30*	*−1.30*	*11.41*	*20.91*	*9.33*	*2.37*	*8.83*
SMB	0.0596	−0.0043	0.0077	0.1236	0.0100	0.0784	0.2158	0.1275	0.0112	0.0703
	3.05	*−0.10*	*0.26*	*4.21*	*0.18*	*1.41*	*5.33*	*5.03*	*0.69*	*3.27*
HML	0.0198	0.0398	0.0491	0.0376	0.0449	−0.0133	−0.0450	0.0243	0.0241	−0.0214
	0.66	*1.10*	*2.35*	*0.92*	*0.84*	*−0.24*	*−1.11*	*0.62*	*0.80*	*−0.78*
UMD	−0.0346	0.0573	0.0199	−0.0045	0.0812	0.0227	0.0248	−0.0018	0.0037	0.0657
	−2.56	*2.18*	*1.28*	*−0.28*	*2.20*	*0.61*	*1.64*	*−0.13*	*0.41*	*4.72*
ar(1)	0.6110	−0.0609	0.1376	0.4436	−0.0630	0.3325	0.1638	0.2771	0.3786	0.3182
	7.17	*−0.93*	*1.61*	*4.73*	*1.04*	*4.88*	*2.10*	*3.85*	*4.41*	*2.53*
R^2	0.50	0.04	0.03	0.55	0.05	0.57	0.87	0.65	0.28	0.59
Dep. Var.	Health care	Long-short	Macro	Market neutral	Merger	Multistrat	Short bias	Techno	Mean	*gi*
C	0.7865	0.3802	0.3149	0.1805	0.3327	0.4667	0.3556	0.4518	0.3214	0.3266
	3.35	*3.59*	*3.67*	*3.30*	*4.62*	*3.62*	*2.79*	*3.62*	*2.50*	*3.22*
mkt_rf	0.4459	0.3122	0.1852	0.0747	0.1056	0.1113	−0.7459	0.5023	0.1635	0.3316
	7.23	*18.31*	*6.87*	*5.52*	*4.87*	*5.08*	*−24.94*	*17.14*	*8.58*	*20.98*
SMB	0.6029	0.1504	0.0726	0.0061	0.0496	0.0672	−0.4232	0.2714	0.0832	0.1205
	3.57	*6.98*	*2.28*	*0.46*	*3.49*	*4.38*	*−10.88*	*7.12*	*3.48*	*6.03*
HML	−0.4781	−0.0827	−0.0103	0.0151	0.0170	0.0080	0.2881	−0.4902	−0.0318	−0.0282
	−4.42	*−3.58*	*−0.30*	*0.78*	*0.74*	*0.30*	*7.31*	*−12.69*	*2.20*	*−1.31*
UMD	0.1438	0.0645	0.0746	0.1010	0.0022	0.0013	−0.0446	0.0712	0.0360	0.0370
	2.25	*4.70*	*3.36*	*7.86*	*0.19*	*0.12*	*−1.83*	*2.98*	*2.18*	*2.90*
ar(1)	0.1910	0.3079	0.0052	0.1571	0.1380	0.4516	0.2279	0.3483	0.2425	0.3204
	1.72	*4.66*	*0.08*	*2.61*	*1.85*	*4.54*	*3.42*	*5.36*	*3.19*	*4.92*
R^2	0.68	0.74	0.34	0.40	0.35	0.47	0.85	0.79	0.49	0.77

Notes: We regress each strategy's excess return on the factors enumerated in line. *mkt_rf* is the market portfolio (S&P500) excess return. For each explanatory variable, the first line provides the regression coefficient while the second line gives the corresponding t-statistic, written in italics over all strategies is of the same order. The strategies' α's range from a low of 0.06 for currency to a high of 0.78 for health care.

The market risk premium is the most important factor impacting hedge fund strategies. Consistent with the orientation of hedge funds, at 0.33, it is very moderate for *gi*. Equity long, emerging and short bias – with respective market β's equal to 0.60, 0.58 and −0.74 – display the highest exposure to the market. But for several strategies, the β is lower than 0.10 – i.e., CTA, diversified, market neutral, fixed income and convertible.

Many studies have found that mutual funds show a tendency to herd regarding their investments – especially in growth stocks (e.g. Grinblatt *et al.*, 1995; Haiss, 2005). It also seems to be the case in the hedge fund industry. Indeed, SMB is second in order of importance in our estimation of the Fama and French three-factor model. This factor also shows a close link with the market return (Figure 2). For *gi*, the factor loading corresponding to SMB is equal to 0.12. The following strategies display the highest exposure to SMB: health care (0.60), technology (0.27), equity long (0.21) and short bias (−0.42). In other respects, hedge funds are less exposed to HML than SMB, the estimated coefficient associated with HML in the *gi* equation being equal to −0.03, which corresponds to the average computed over all strategies. Hedge funds thus seem to prefer growth stocks over value stocks (Grinblatt *et al.*, 1995; Haiss, 2005). However, some strategies are highly exposed to HML – i.e., technology (−0.49), health care (−0.47), and short bias (0.28). Finally, hedge funds show a tendency to follow momentum-based strategies. The hedge fund general index has an exposure of 0.04 to UMD. The strategies having the highest exposure to UMD are heath care (0.14), market neutral (0.10) and technology (0.07).

4.2 An augmented version of the five-factor model encompassing the Fung and Hsieh factors

To study the interactions between factors in the Fama and French five-factor model, we augment it with the seven factors proposed by Fung and Hsieh (1997, 2001, 2004) to account for the dynamic dimensions of hedge fund strategies. These factors comprise five categories of lookback straddles, the change in the ten-year interest rate, and the change in the credit spread. The lookback straddles[35] – which are especially useful to study the trend followers – are: the bond lookback (bond_look), the stock lookback (stock_look), the short-interest lookback (shortint_look), the currency lookback (currency_look) and the commodity lookback (commod_look). Following the addition of these seven factors, our return model takes the following form:

$$
\begin{aligned}
R_{it}-r_{ft} = {} & \alpha+\beta_1\left(R_{mt}-r_{ft}\right)+\beta_2 SMB_t+\beta_3 HML_t+\beta_4 UMD_t+\beta_5 CMA_t+\beta_6 RMW_t \\
& +\beta_7 bond\_look_t+\beta_8 stock\_look_t+\ldots+\beta_9 shortint\_look_t+\beta_{10} currency\_look_t \\
& +\beta_{11} commod\_look_t+\beta_{12} d(CredSpr_t)+\beta_{13} d(10Y_t)+\beta_{14} ar(1)+\varepsilon_{it} \qquad (7)
\end{aligned}
$$

where $d(CredSpr)$ stands for the change in the credit spread – i.e., the spread between the BBB and AAA US corporate bond yields, and $d(10Y_t)$ is the change in the rate of the ten-year US federal government bond.

4.2.1 The impact of the seven Fung and Hsieh' factors. Table IV provides the estimation of Equation (7) over on our sample of hedge fund strategies' returns. To run our regressions, we rely on the stepwise least-squares algorithm. Only the constant term and the market risk premium are added initially in the regressions. The other

Dep. Var.	Convertible	CTA	Currency	Distressed	Diversified	Emerging	Equity long	Event driven	Fixed income	Funds of funds
C	0.3938	0.1717	0.0434	0.3733	0.4034	0.1804	0.3587	0.3858	0.2673	0.1767
	2.68	*1.44*	*0.50*	*3.45*	*2.12*	*0.67*	*3.66*	*4.10*	*3.12*	*2.18*
mkt_rf	0.0769	0.0628	0.0514	0.1676	0.0685	0.5338	0.5577	0.2390	0.0630	0.1805
	4.69	*2.18*	*2.90*	*9.27*	*1.47*	*12.29*	*29.58*	*14.51*	*4.06*	*11.63*
SMB	0.0569			0.1198		0.1433	0.2237	0.1196		0.0784
	2.77			*5.20*		*2.40*	*9.92*	*5.71*		*4.34*
HML										
UMD	−0.0465	0.0396			0.0591					0.0627
	−3.48	*1.82*			*1.88*					*5.47*
CMA		0.1326	0.1164		0.1846	−0.2408	−0.1531			−0.0662
		2.32	*3.14*		*2.15*	*−2.85*	*−4.06*			*−2.22*
RMW						0.1400				
						1.74				
Bond look		0.0280		−0.0224	0.0380			−0.0160		
		3.39		*−4.15*	*2.96*			*−3.27*		
Commodity look		0.0253			0.0456					
		2.95			*3.49*					
Currency look		0.0363	0.0361		0.0441		0.0113		−0.0097	
		5.27	*8.72*		*4.41*		*2.64*		*−2.54*	−0.0140
Short-interest look	−0.0131			−0.0096	0.0000	−0.0267	−0.0111	−0.0076	−0.0132	*−5.69*
	−4.64			*−3.12*		*−4.01*	*−3.69*	*−2.73*	*−4.89*	0.0086
Stock look					0.0266					*1.78*
					2.42					0.0000
d(taux 10 ans)				1.2357			0.6325	0.7601		
				3.33			*1.76*	*2.26*		−2.7700
d(credit spread)	−5.0714			−4.6700		−3.4000	−2.0100	−1.9300	−5.4500	*−4.96*
	−6.68			*−6.39*		*−2.13*	*−2.89*	*−2.95*	*−8.78*	0.2073
ar(1)	0.4797	0.0219	0.1174	0.2607	−0.0053	0.3410	0.2065	0.2316	0.1925	*2.87*
	7.09	*0.31*	*1.73*	*3.75*	*−0.08*	*5.20*	*2.96*	*3.31*	*2.80*	0.68
R^2	0.63	0.29	0.29	0.68	0.32	0.62	0.89	0.70	0.54	

(continued)

Table IV. Estimation of the Fama and French's augmented five-factor model

Table IV.

Dep. Var.	Health care	Long-short	Macro	Market neutral	Merger	Multistrat.	Short bias	Techno	Mean	*gi*
C	1.1024	0.3809	0.3944	0.2273	0.3040	0.4604	0.3316	0.6370	0.3663	0.3563
	4.49	*3.57*	*4.09*	*4.15*	*4.72*	*5.72*	*2.09*	*3.72*	*3.14*	*4.02*
mkt_rf	0.3249	0.3047	0.2116	0.0724	0.0927	0.0994	−0.7442	0.4385	0.1556	0.3022
	6.98	*17.51*	*10.23*	*6.75*	*7.89*	*8.09*	*−25.95*	*15.57*	*10.64*	*19.23*
SMB	0.3507	0.1499	0.0680			0.0615	−0.4112	0.1789	0.0633	0.1272
	5.52	*6.90*	*2.61*			*3.86*	*−11.32*	*4.73*	*3.63*	*6.92*
HML		−0.0876					0.2928	−0.2972	−0.0051	
		−3.78					*7.60*	*−5.76*	*0.95*	
UMD	0.1933	0.0620	0.0637	0.0974			−0.0509	0.0743	0.0308	0.0332
	5.85	*4.52*	*3.89*	*11.39*			*−2.21*	*3.63*	*2.45*	*2.83*
CMA	−0.2198							−0.2011	−0.0249	−0.1007
	−2.48							*−2.88*	*1.23*	*−3.27*
RMW	−0.7823							−0.2817	−0.0513	
	−9.40							*−4.78*	*0.88*	
Bond look									0.0015	
									0.77	
Commodity look			0.0129						0.0047	
			1.99						*0.47*	
Currency look			0.0218					0.0150	0.0086	
			4.33					*2.62*	*1.70*	
Short-interest look		−0.0056	−0.0102	−0.0054	−0.0073	−0.0117			−0.0075	−0.0106
		−1.99	*−2.94*	*−2.93*	*−3.47*	*−5.46*			*2.53*	*−4.35*
Stock look			0.0186	0.0095	−0.0094				0.0030	
			2.77	*2.66*	*−2.27*				*0.66*	
d(taux 10 ans)									0.1460	
									0.41	
d(credit spread)						−3.0400			−1.5745	−2.0800
						−5.52			*2.24*	*−3.52*
ar(1)	0.2454	0.3079	0.0704	0.1237	0.1495	0.3041	0.2278	0.3473	0.2127	0.2778
	3.66	*4.66*	*1.00*	*1.78*	*2.19*	*4.15*	*3.42*	*5.27*	*3.12*	*3.94*
R^2	0.72	0.74	0.43	0.42	0.38	0.59	0.86	0.83	0.58	0.80

Notes: The Fama and French's augmented five-factor model includes the Fama and French five-factor model (Fama and French, 2015) to which we add the momentum factor UMD and the seven Hsieh's factors (Fung and Hsieh, 1997, 2001, 2004). Except for the market excess return, each factor is introduced in each regression using the stepwise least-squares. For each explanatory variable, the first line provides the regression coefficient and the second line, the *t*-statistic written in italics variables are introduced progressively in order of importance insofar as their significance level exceeds a threshold of 90 percent. The average adjusted R^2 computed over the strategies increases from 0.49 to 0.59 when shifting from the three-variable model to the augmented five-factor model. The increase in the R^2 is especially important for some strategies like CTA, currency, diversified, convertible and fixed income. This better performance is essentially due to the addition of the lookback straddles and the change in the credit spread in the three-factor model. Surprisingly, the average α does not decrease in the augmented five-factor model compared to the three-factor one, its respective level being 0.36 and 0.32. As assessed by Fama and French (2015), "if an asset pricing model completely captures expected returns, the intercept is indistinguishable from zero[36]." In this sense, at least for hedge funds, the augmented five-factor model is not a "more complete" model than the three-factor model.

Before examining the interactions between the Fama and French factors, we look at the behavior of the Fung and Hsieh factors. The importance of the lookback straddles vary substantially from one strategy to the next. The lookback which impacts the most strategies is the short-interest lookback. In total, 12 strategies display a significant negative exposure to this variable. Incidentally, it is the only kind of straddle to which the hedge fund weighted composite index is significant. The negative sign of the short-interest straddle may be explained by looking at the plot of this variable (Figure 4). We note that this straddle peaks at the major crises – i.e., the 1998 Asian-Russian-LTCM crisis, the 2000 tech-bubble crisis and the 2007-2009 subprime crisis – and these peaks dominate the other observations on this time series. This variable is thus more an indicator of economic uncertainty than an indicator of the yield on short-interest straddles. The reaction of strategies' returns to this variable signals the drop of these returns during crises. In this respect, Figure 5 shows the Kalman filter estimate of the time-varying exposure to the short-interest lookback of three strategies which were particularly hit by the subprime crisis – i.e., fixed income, multistrategy and funds of funds. We note an important increase in the exposure of these three strategies to the short-interest lookback during the subprime crisis, which signals the big drop in their returns. Figure 5 also plots the Kalman filter estimate of the time-varying market β for these three strategies. The β of the

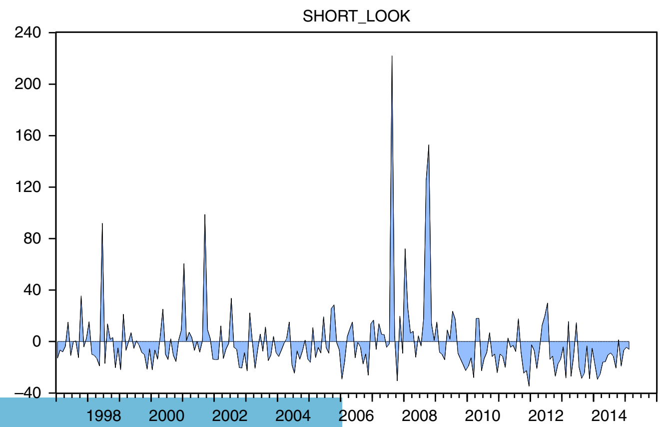
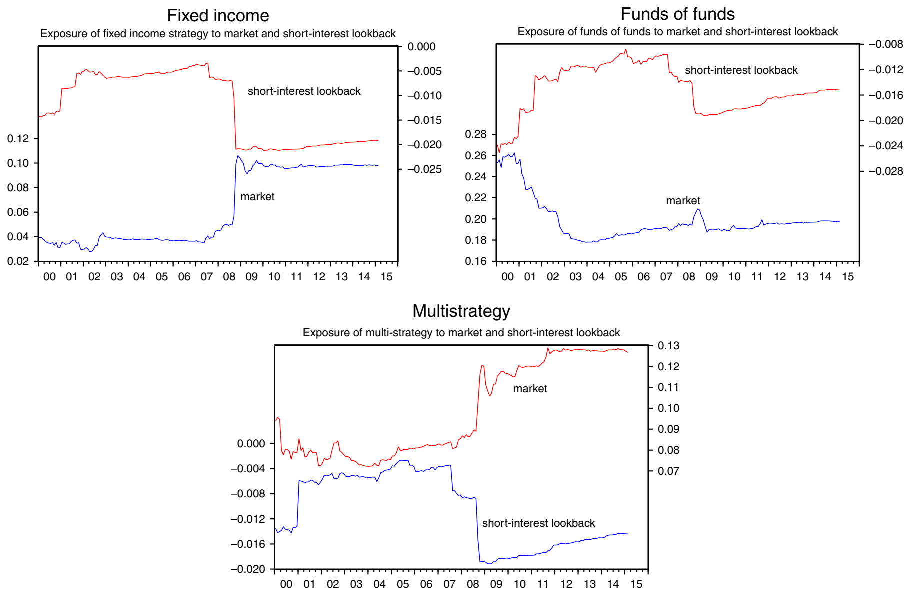


Figure 4.
Short-interest lookback return



Note: The time-varying coefficients have been computed using the Kalman filter algorithm

Figure 5. Time-varying exposures of several strategies to market and short-interest lookback

multistrategy and the fixed income strategy also increased substantially during the subprime crisis, an obvious rise in systematic risk.

Trend followers should have a positive exposure to lookbacks. In this respect, six strategies have a significant positive exposure to the currency lookback while one of them – fixed income – is negatively exposed to this variable. This may be related to sovereign debt problems faced by some European countries. Four strategies also display a significant positive exposure to stock lookback while one, merger, has a significant negative exposure. Moreover, three strategies – i.e., CTA, diversified and macro – have a significant positive exposure to the commodity lookback. Distressed and event-driven strategies suffer from the volatility in the bond market as measured by the bond lookback. Finally, in addition to the general index, nine strategies have a significant negative exposure to the change in the credit spread. Credit risk is thus omnipresent in the hedge fund industry. The fixed income strategy was particularly exposed to credit risk during our observation period since mortgage-backed securities were a significant share of assets in this category during the subprime crisis.

4.2.2 The interactions between the five Fama and French's factors. We now turn to the study of the interactions between the Fama and French factors. More precisely, we want to test three conjectures:

(1) Conjecture 1: the HML factor is redundant in the presence of CMA and RMW, i.e., the risk embedded in HML is captured by CMA and RMW;

(2) Conjecture 2: HML interacts more with CMA than with RMW, i.e., firms with a high book-to-market ratio tend to invest less; and

(3) Conjecture 3: SMB interacts more with RMW than with CMA, i.e., small firms tend to be less profitable than big ones.

First, note that the seven Fung and Hsieh's factors do not interact significantly with the five Fama and French factors so we can compare directly the factor loadings in Table III – i.e., the three-factor model – to their corresponding values in Table IV, i.e., the augmented five-factor model. Second, when shifting from the three-factor to the five-factor model, we observe a decrease in the mean of the market βs computed over all strategies – from 0.33 to 0.16 – even if the market β of the hedge fund general index remains close to 0.30. The decrease in the mean of the market βs is essentially due to two strategies: health care and to a lesser extent technology. When shifting from the three-factor to the five-factor model, the market β of health care drops from 0.44 to 0.32 while the market β of technology decreases from 0.50 to 0.44. The other strategies' βs do not significantly change. The drop in the market βs when transitioning from the three-factor to the five-factor model seem to be associated with the interaction between the market risk premium and SMB (Figure 2). In this respect, the mean market β computed over all strategies is equal to 0.1635 in the three-factor model (Table III) and to 0.1556 in the five-factor model (Table IV), i.e., a 4.8 percent drop.

Let us first examine conjecture 1 – i.e., the redundancy of HML in the presence of CMA and RMW. To simplify the analysis, we have pooled in Table V the exposures of the strategies to either SMB or HML in the three-factor model when these exposures are statistically significant. For each of these strategies, we perform simple comparisons of

Table V. Alternative specifications of the Fama and French model

	SMB	HML	CMA	RMW	R^2
3-factor model					
Augmented 5-factor model					
gi	0.12**	−0.03*			0.77
	0.13**		−0.10**		0.80
CTA	0.00	0.04			0.04
			0.13**		0.29
Currency	0.01	0.05**			0.03
			0.12**		0.29
Distressed	0.12**	0.04			0.55
	0.11**				0.68
Equity long	0.22**	−0.04			0.87
	0.22**		−0.15**		0.90
Event driven	0.13**	0.02			0.65
	0.11**				0.71
Funds of funds	0.07**	−0.02			0.59
	0.07**		−0.07**		0.67
Health care	0.60**	−0.47**			0.68
	0.35**		−0.21**	−0.78**	0.71
Long-short	0.14**	−0.08**			0.73
	0.11**		−0.10**	−0.13**	0.75
Multistrategy	0.06**				0.47
	0.06**				0.59
Short bias	−0.41**	0.29**			0.85
	−0.36**		0.23**	0.24**	0.84
Techno	0.27**	−0.49**			0.79
	0.17**	−0.29**	−0.20**	−0.28**	0.83

Notes: For each strategy, the first line reports the coefficients of Table III, and the second line, the coefficients of Table IV. *,**Significant at 10 and 5 percent levels, respectively these exposures to the ones of the four factors associated with firms' characteristics in the five-factor model – i.e., SMB, HML, CMA and RMW.

The strategies most linked to the HML factor in the three-factor model are technology and health care, their respective exposures being −0.49 and −0.47. In the five-factor model, for technology, the weight of HML is shared between CMA and RMW – the respective exposures being −0.20 and −0.28. However, the coefficient of HML remains negative and high at −0.29. For health care, the weight of HML is shifted to CMA (−0.21) and mainly to RMW (−0.78). For long-short and short bias, we observe the same weight redistributions of the loading of HML between CMA and RMW in the augmented five-factor model. Therefore, these results show that CMA and RMW are very good substitutes for HML but that HML is not made redundant by the two new Fama and French's factors. This seems to invalidate conjecture 1.

Moreover, conjecture 2 is not always true as suggested by our findings – especially in the case of health care where RMW inherits a greater share of the HML loading. However, this conjecture is verified in many cases. For example, for several strategies – i.e., *gi*, CTA, currency, equity-long – the loading of HML in the three-factor model is only shifted to CMA in the augmented five-factor model[37].

Conjecture 3 is more easily verified. For the general index and many strategies – i.e., distressed, equity-long, event driven, funds of funds and multistrategy – the positive share of SMB is strictly recuperated by RMW in the stepwise least-squares, which takes a negative sign. For hedge funds, consistent with the stylized facts, the SMB factor plays a role which is similar to −RMW (or WMR): the return of small firms substitute to the return of weak firms in the strategies' equations.

4.2.3 Robustness checks: introducing interaction terms in the augmented five-factor model. In this section, we provide further evidence on the relationship between SMB and HML, on the one hand, and CMA and RMW, on the other hand, by introducing interaction terms in the augmented Fama and French model, whenever possible. Indeed, to define an interaction term between two groups of factors – traditional and new ones – the factors associated with the interaction term must be both significant in the regression, which obviously restricts the field of possibilities. Table VI provides further evidence of the interaction between SMB and RMW, as measured by SMB × RMW. This interaction term is significant for several strategies – i.e., health care, long-short and short-bias. For short bias, the interaction term SMB × CMA is also

Table VI.
Interaction terms and the augmented Fama and French model

	SMB × CMA	SMB × RMW	HML × CMA	HML × RMW
Emerging	0.021	−0.006		
	1.00	*−0.79*		
Funds of funds	0.005			
	0.60			
Health care	−0.007	−0.035		
	−0.32	*−4.76*		
Long-short	0.009	−0.009		
	1.03	*−2.99*		
Shor-bias	−0.028	0.018		
	−1.81	*3.31*		
Techno	0.001		0.025	0.017
	0.06		*2.35*	*2.85*

Note: The interaction terms are introduced in the augmented five-factor model (Table IV) significant at the 10 percent level. Finally, for technology, CMA and RMW both interact with HML but the coefficient of HML × CMA (0.025) is higher than the coefficient of HML × RMW (0.017), which suggests that CMA is closer to HML than RMW.

5. Conclusion

The *q*-factor model is an attractive one because, in contrast to factor models based on "anomalies" like the Fama and French (1993) model, it lies on robust theoretical foundations. It is thus important to study the interactions between the conventional Fama and French (1993) factors – i.e., SMB and HML – and the new Fama and French (2015) factors explicitly based on the *q*-factor model – i.e., CMA and RMW – to highlight the theoretical relevance of the Fama and French three-factor model. Since we performed this empirical study using a comprehensive hedge fund database managed by BarclayHedge, we can also better characterize hedge fund strategies with the help of the new factors.

Our findings show that CMA and RMW are jointly good substitutes for HML, albeit they are not perfect substitutes. Most hedge fund strategies are negatively exposed to HML, which corresponds, according to our estimations, to a negative exposure to both CMA and RMW. Hedge funds tend to prefer growth stocks to value stocks, or, in the *q*-space, they prefer to put their money in firms with a high investment-to-assets ratio and which are relatively weak[38]. Note that theoretically, a negative exposure to HML is compatible with a negative exposure to CMA but a positive exposure to RMW, not a negative one like in our experiments. Fama and French have also confronted this apparent puzzle in their sample. The joint presence of these two characteristics – negative CMA and negative RMW – could be associated with firms issuing growth stocks (e.g. high-tech and health care), which, in a dynamic approach – i.e., a conditional setting – may make big investments in the early stages of their life and which tend to be more profitable later. Alternatively, they may overinvest in an attempt to reduce their financial distress. More precisely, they hope to generate enough positive cash-flows with their investments to relax their financial constraints (e.g. Fazzari *et al.*, 1988; Thomas and Worrall, 2014).

According to our results, SMB is closer to RMW than HML, and HML is closer to CMA – i.e., small firms tend to be weak and firms issuing value stocks tend to be conservative regarding their investments. In this respect, the conditional correlations between SMB and CMA, on the one hand, and HML and RMW, on the other hand, are quite unstable and may change signs. Moreover, in line with the theoretical *q*-factor model, the correlation between HML and RMW has tended to be negative since the start of the subprime crisis, suggesting that value stocks have been associated with weak firms since this period (or growth stocks tend to be more valuable).

Overall, our results indicate that the presence of SMB and HML is justified in empirical asset pricing models. Even if, theoretically, CMA and RMW should be sufficient to explain stock returns[39], they are only proxies for the corresponding theoretical values because of measurement errors. Insofar as SMB and HML help span the universe of risks to which stocks are confronted, they continue to have their place in empirical return models.

Notes

1. Tobin's *q* is the ratio of the market value of capital to its replacement cost. When a firm's cost of capital decreases, the market value of capital increases (i.e. *q* increases), which fosters firm's real investment – i.e., a capital flow. A central bank can impact firms' cost of capital via its control over the short-term interest rate.
2. That is, a marginal unit of capital.

3. Indeed, in the simple Gordon's formula, $r = (D_1/P) + g$, where r is the stock return; D_1 is firm's dividend a time 1; P is the stock price; and g is the rate of growth of D. Therefore, an increase in D_1 – i.e., an increase in profitability – is associated with an increase in r.
4. As assessed by Keynes (1936) in his "General theory of employment, interest and money", Fisher (1930), in his "Theory of interest", had previously developed the relationship between the cost of capital and investment without naming it. Keynes labeled it: "the marginal efficiency of capital".
5. In addition, factors accounting for the option-like (dynamic) dimensions of hedge fund strategies must be added in the hedge fund empirical return model (Fung and Hsieh, 2004).
6. However, Fama and French (2015) have found *ex post* some theoretical justifications for their three-factor model.
7. CMA is the abbreviation of "conservative minus aggressive" – i.e., a portfolio which is long in stocks of firms with a low ratio of investment-to-assets and short in stocks of firms with a high ratio of investment-to-assets.
8. RMW is the abbreviation of "robust minus weak" – i.e., a portfolio which is long in stocks of robust firms in terms of profitability and short in stocks of weak firms in terms of profitability.
9. SMB is the abbreviation for "small minus big" – i.e., a portfolio which is long in stocks of small firms and short in stocks of big firms.
10. HML is the abbreviation for "high minus low" – i.e., a portfolio which is long in stocks of firms having a high book-to-market ratio and short in stocks of firms having a low book-to-market ratio.
11. Moreover, as argued by Fama and French (2015, p. 3), Titman *et al.* (2004) and Novy-Marx (2013) assess that the three-factor model is an incomplete model for factor returns since it misses much of the variation in average returns related to profitability and investment.
12. The study of Fama and French (2015) was performed on industries' cross-sectional longitudinal data. This analysis was quite static. In this paper, we transpose their approach to a time series analysis of hedge fund managed portfolios. Our results should thus differ from those of Fama and French since we cast our analysis in the context of hedge fund portfolio management using the q-factors.
13. Our sample period is long enough to include many major crises like the Asian-Russian-LTCM crisis (1998), the tech-bubble crisis (2000-2002), the subprime crisis (2007-2009) and the European sovereign debt crises (2010-2012). Indeed, it is during crises that the strategies of hedge fund are the most dynamic (Racicot and Théoret, 2016a, c).
14. More precisely, the Fung and Hsieh's seven factors capture the dynamic aspects of hedge fund strategies.
15. That is, the short-sellers strategy, a strategy which takes advantage of declining stocks. This strategy has a negative market β.
16. That is, a negative CMA.
17. That is, a negative RMW.
18. In its original formulation, the scaling factor of investment is k_{it} – i.e., firm's i capital – and not firm i's assets as in Equation (1) (Cochrane, 1991, 2011). The Euler equation then reads as (see Equation (10), Cochrane, 2011):

$$1 + a\frac{I_{it}}{k_{it}} = E_t[M_{t+1}\pi_{it+1}] = q_t$$

Insofar as the scaling factor only affects the adjustment cost of capital, the two formulations are equivalent. This alternative formulation of the q-theory directly shows that q_t is the marginal benefit of investment, equal to the present value of the expected cash-flow of one unit of capital.

19. Fama and French (2015) do not introduce the momentum factor (UMD) proposed by Carhart (1997) and the liquidity factor proposed by Pástor and Stambaugh (2003) in their new asset pricing model. They justify this omission by the fact that these two factors have regression slopes close to zero in their experiments so they decided to discard them. According to Fama and French (2015), these factors produce trivial changes in model performance. However, in our setting, UMD is often significant – as many hedge funds follow momentum-based strategies – so we include this factor in our estimations. Moreover, we account for illiquidity with an autoregressive variable.

20. Equation (4) is a simplification of the q-theory. According to this theory:

$$1+\alpha\frac{I_{it}}{k_{it}} = \left(\frac{\text{market value}}{\text{book value}}\right)_t = q_t,$$

q_t being equal to: $E_t[M_{t+1}\pi_{i,t+1}]$ (see Cochrane, 2011).

21. Note that investment represents a decrease in the book value of equity (B_t) – i.e., an expense or a negative cash-flow. Instead of putting dB in Equation (4), Miller and Modigliani (1961) put I in their original equation – i.e., Equation (9) in their 1961 article:

$$V(0) = \sum_{t=0}^{\infty}\frac{X(t)-I(t)}{(1+\rho)^{t+1}}.$$

22. Fama and French (2015) even argue that Equation (4) is a tautology since it is derived from the Gordon-Shapiro equation, so it has a great generality.

23. Consistent with the original formulation of the q-model, we scale I_{it} by k_{it} in this equation, and not by A_{it} as in Equation (1). Since we scale investments by capital rather than by assets in this equation, we thus adjust the coefficient of this ratio from a in Equation (1) to b in Equation (5). This change allows us to equalize directly the RHS of this equation to Tobin's q.

24. Since the LHS of Equation (5) is equal to Tobin's q.

25. This interpretation is based on the famous Euler equation in asset pricing, which establishes a link between the price of an asset and the stochastic discounted value of its cash-flows. The stochastic discount factor M_{t+1} is proportional to $u'(c_{t+1})/u'(c_t)$ (Cochrane, 2005).

26. See for instance Lhabitant (2001), Straumann (2003), Gregoriou and Pascalau (2011) and Brown *et al.* (2012) for papers using Barclay's data. As reported by Straumann (2003), issues encountered in the Barclay database are shared by other databases. In this respect, returns reported by some strategies – i.e., macro – are more reliable than the ones provided by others like CTA. These latter strategies are associated with hedge funds dealing with illiquid securities and which are thus involved in return smoothing. The returns of their portfolios tend to be highly autocorrelated.

27. The sample used by Fama and French (2015) runs from July 1963 to December 2013. It includes all NYSE, AMEX and NASDAQ stocks drawn from the CRSP and COMPUSTAT databases. To perform their tests, they build diversified portfolios of these stocks using various combinations of factors and various quintiles within the factor categories. They reproduce the methodology used in Fama and French (1993) in order to see whether the five-factor model better explains stocks excess returns than the three-factor model.

28. The address of French's website is: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. The CMA and RMW factors were recently added to the French's database.
29. The address of Hsieh's database is: https://faculty.fuqua.duke.edu/~dah7/HFData.htm
30. In this respect, Calmès and Théoret (2015) rely on an MGARCH approach to document the behavior of the conditional correlations between indicators of bank performance and bank product-mix over the business cycle. They find that the MGARCH performs well to capture the dynamic dimensions of correlations. Moreover, Sabbaghi (2012) and Racicot and Théoret (2016c) show how cross-sectional conditional covariances and correlations are related to the dynamic behavior of risk.
31. Our multivariate GARCH regressions indicate that the coefficients used to compute the conditional correlation between SMB and RMW, on the one hand, and HML and CMA, on the other hand, are very significant. These conditional correlations are persistent through time in contrast to the other correlations provided in Figure 1.
32. Note that the majority of the coefficients associated with the multivariate GARCH used to compute this correlation are significant. We thus may infer from the change in the sign of the correlation over the sample period that growth stocks were less profitable than value stocks between 1997 and 2006. The Asian-Russian-LTCM crisis and especially the tech-bubble crisis were damageable for this kind of stocks. Figure 1 indicates that growth stocks have recovered since 2007. The negative theoretical relationship between HML and RMW is only a long-term relationship since it is computed using expected returns and not realized ones.
33. Using another large database – i.e., the Greenwich Alternative Investment database – we also find the same investment profile for hedge funds (Racicot and Théoret, 2016b).
34. There are more sophisticated ways to account for return smoothing (see Getmansky *et al.*, 2004 and Brown *et al.*, 2012).
35. A lookback call option gives the right to buy the underlying asset at its lowest price observed over the life of the option. Similarly, a lookback put option allows the owner to sell the underlying asset at the highest price observed over the life of the option. The combination of these two options is the lookback straddle (Fung and Hsieh, 2001). Straddles are useful for volatility trading and hedging (Hull 2015).
36. According to Cochrane (2011), there is no α. There are just β's which are taken into account and β's which are not accounted for, question of ignorance.
37. For more evidence, see Table V.
38. Using another large database – i.e., the Greenwich Alternative Investment database – we also find the same investment profile for hedge funds (Racicot and Théoret, 2016b).
39. We neglect here the problems presented by the covariance between the stochastic discount factor and firms' cash-flows – i.e., the risk premium in the q-factor model (see Equation (5)).
40. We thank an anonymous referee for his suggestion to add this section.

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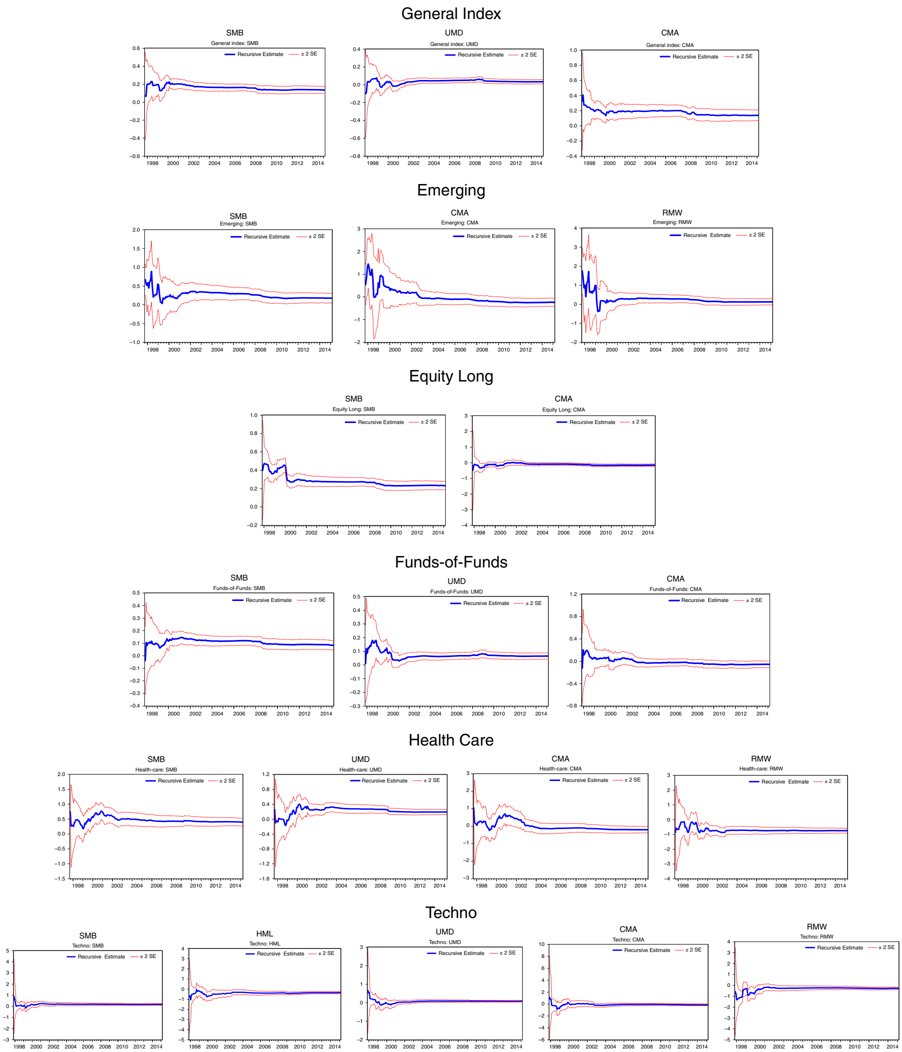
Appendix. Stability tests

As a robustness check, we examine the stability of our results[40]. Indeed, if our results were unstable, they could be due to chance, and in this case, they would be obviously less interesting. Figure A1 reports the recursive coefficients of the hedge fund general index and of key strategies which have a significant exposure to the q-factors – i.e., CMA and/or RMW. These strategies are: emerging, equity long, funds of funds, health care and techno. We also report the recursive coefficients of SMB, HML and UMD when they are significant at the 5 percent level.

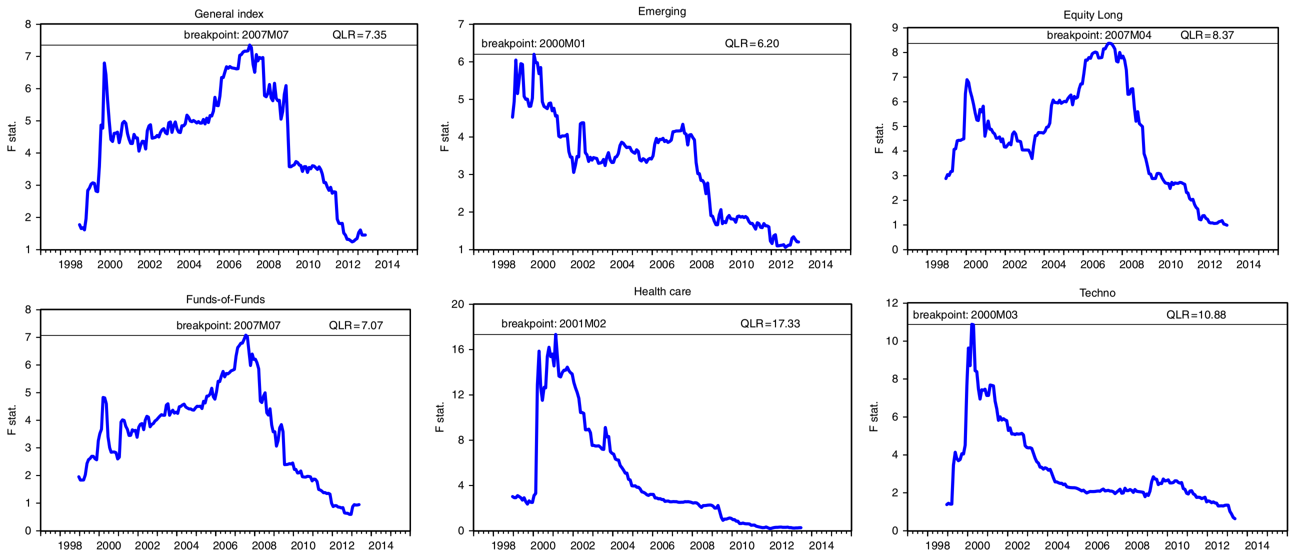
Since 2000, the estimated coefficients of the factors have been usually very stable for the general index and for the strategies considered. The computed confidence intervals for the factors are also very tight. We find that the emerging and health care strategies have reduced their exposure to CMA from 1997 to 2002 but the recursive coefficients for this factor are very stable thereafter for both strategies. We also note that the estimated recursive coefficients for the techno strategy, which is significantly exposed to the five factors considered, have been especially stable since 2000 and that their confidence intervals are very tight.

Figure A2 provides the Quandt-Andrews unknown breakpoint tests for the general index and for the strategies analyzed in Figure A1 (Quandt, 1960; Andrews, 1993, 2003; Stock and Watson, 2003, 2011). The behavior of the F-statistic associated with the test shows that the general index and the equity long strategy display a significant breakpoint during the subprime crisis. The F-plot of the funds of funds strategy reveals a breakpoint only during the subprime crisis. The other strategies – i.e., emerging, health care and techno – display a significant breakpoint only during the tech-bubble crisis. This result is consistent with the behavior of the recursive estimates of the emerging and health care strategies around the tech-bubble crisis. This change in the behavior of hedge funds during crises is normal since the strategies followed by financial institutions, including hedge funds, tend to be revised during crises compared to normal times (expansion periods). There is, thus, an obvious asymmetry in the behavior of financial institutions according to the phase of the business cycle (e.g. Calmès and Théoret, 2014; Racicot and Théoret, 2016a, c).



Note: The two dashed lines enclose the 95 percent confidence interval for the estimated coefficient

Figure A1.
Stability tests:
recursive estimates



Notes: According to Stock and Watson (2011, p. 560), the Quandt likelihood ratio (QLR) statistic is given by: $QLR = MAX[F(\tau_0), F(\tau_0+1),\ldots,F(\tau_1)]$ $\tau_0 \leqslant \tau \leqslant \tau_1$ where $F(.)$ refers to the standard F-statistic evaluated at time τ. In other words, the QLR statistic is the maximum F-statistic computed over a possible set of breakpoints stretched over the sample used

Source: For more detail on the Quandt-Andrews test, see Quandt (1960), Andrews (1993, 2003), and Stock and Watson (2003, 2011)

Figure A2. Quandt-Andrews unknown breakpoint tests